Hilbert-Huang Transform Analysis of Intradecadal Variations in the Length of Earth's Day

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Abstract

Intradecadal periods in the length of Earth's day (ΔLOD) reveal angular momentum exchanges as a result of processes in the Earth's core. The properties of two previously identified intradecadal oscillations in the ΔLOD signal with periods of approximately 6 and 8.5 years, respectively, are investigated from 1962 to 2025, using the Hilbert-Huang transform method. Initially, angular momentum contributions from zonal tides and the Earth's atmosphere are removed, before the application of Butterworth bandpass filters to isolate the oscillations. The two intradecadal oscillations are found to have periods of 5.8 ± 0.8 years and 9.1 ± 1.6 years, respectively. The six-year oscillation shows a decaying trend from 2010 onwards, associated with a shortening in the period of oscillation. The eight-year oscillation breaks down between 1995 and 2000, which may be linked with geomagnetic jerk activity.

Introduction

Variations in the rotation rate of the Earth, resulting in changes in the length of Earth's day (ΔLOD), occur due to angular momentum exchanges from mass redistribution and motion in the Earth's fluid envelope and liquid core (Rekier *et al.* 2022). After the removal of the effects of tidal forces and atmospheric angular momentum excitations, several remaining long-term oscillations in ΔLOD can be recognised and attributed to core processes. Two notable intradecadal periods, and the subject of this study, are a 5.9-year period (Abarca del Rio *et al.* 2000; Holme *et al.* 2013), known henceforth as the six-year oscillation (SYO), and an 8.5-year period (Ding 2019; Rosat *et al.* 2023), known hereafter as the eight-year oscillation (EYO). The mechanisms involved in the generation and formation of both the SYO and the EYO remain topics of ongoing research, with a range of models proposed to describe the presence of the two oscillations. The SYO has been proposed to be related to fluid flow at the surface of the outer core (Istas *et al.* 2023; Rosat *et al.* 2023), or to mantle-inner core gravitational coupling modes (Mound *et al.* 2003; Duan *et al.* 2020b). The generation of the EYO has been ascribed to either quasi-geostrophic magneto-Coriolis waves (Gillet *et al.* 2022), or torsional core oscillations (Duan *et al.* 2020a).

Previous studies have used methods such as the normal Morlet wavelet transform (Duan *et al.* 2020a), cubic splines (Madsen *et al.* 2025) and the continuous wavelet transform (Rosat *et al.* 2023) as a means of investigating intradecadal periods in the ΔLOD signal. This study will compare previous findings with those of an alternative method of examination, the Hilbert-Huang transform (HHT). The HHT method allows for the investigation of the oscillatory components and instantaneous frequency (IF) trends of a signal over time, facilitating a greater understanding of the structures and properties of the SYO and EYO. We use the Rayleigh criterion to define upper and lower limits for bandpass filtering in order to isolate the SYO and EYO, respectively, before applying the HHT method to examine and provide information on the nature and behaviour of the signals.

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Data

The period of analysis for this study spans from 01/01/1962 to 01/05/2025. This work uses the EOP 20 C04 ΔLOD series (Bizouard *et al.* 2009), which is a combined series comprising measurements of ΔLOD from a variety of space-geodetic techniques (Gross 2015). The EOP 20 C04 series and fluid envelope angular momentum excitation functions used are provided by the International Earth Rotations and Reference Systems Service (IERS). Between 01/01/1962 and 31/12/1975, the excitation functions computed by Atmospheric and Environmental Research (AER) (Zhou *et al.* 2006) are used. From 01/01/1976 onwards, the model computed by the GFZ Helmholtz-Zentrum für Geoforschung (Dobslaw *et al.* 2010) is used. The angular momentum functions are composed of both mass and motion terms, which are combined for the purpose of this study. The modelling of periodic variations in ΔLOD as a result of zonal tides is conducted as set out by the IERS conventions (Petit *et al.* 2010). This model uses a combination of elastic body tide (Yoder *et al.* 1981), inelastic body tide (Wahr *et al.* 1986), and ocean tide (Kantha *et al.* 1998) models to determine the effects of zonal tides on the Earth's rotation.

Methods

Signal Processing

The contributions from core processes are isolated by subtracting the zonal tidal effects and atmospheric angular momentum functions, in accordance with the method of Mohn et al. (in prep.). To then isolate specific periods, a first-order Butterworth filter is applied in the frequency domain. The Rayleigh criterion is used to select the applied bands, where the criterion dictates, for two periodic components, the smallest measurable frequency separation. For two frequencies, σ_1 and σ_2, this is given by

$$|\sigma_1 - \sigma_2|L > R \tag{1}$$

where L is the recording length of the time series, and R is the Rayleigh constant, with value 1. This can then be used to calculate the Rayleigh periods, P_{min} and P_{max}, which indicate the range of oscillation periods that cannot be distinguished from one another (Pfeffer *et al.* 2023). For a period of interest, P, the Rayleigh periods are

$$P_{\text{min}} = \frac{1}{\frac{1}{P} + \frac{1}{L}} \tag{2}$$

and

$$P_{\text{max}} = \frac{1}{\frac{1}{P} - \frac{1}{L}}. \tag{3}$$

The Hilbert-Huang Transform

The HHT is a two-stage process for the extraction of the oscillatory components and IF trend of a signal over time. The two stages in this process are empirical mode decomposition (EMD) and Hilbert spectral analysis, implemented using the Python package, `emd` (Quinn *et al.* 2021).

Empirical Mode Decomposition

EMD is a method of decomposing a time series function into a small number of oscillatory modes, known as intrinsic mode functions (IMFs) (Huang *et al.* 1998). The structure of each IMF fulfils two conditions (Roberts *et al.* 2007):

1. The difference between the number of extrema and horizontal axis crossings does not exceed a value of one.
2. The average value of the two envelopes of the function, defined by the local maxima and minima, must be zero at every point.

Identified IMFs are extracted until the original signal contains no further oscillatory signals, leaving the residual. Each IMF extracted is the largest remaining frequency mode in the signal. EMD is applied to filtered signals to isolate IMFs of interest.

Hilbert Spectral Analysis

Hilbert spectral analysis uses the Hilbert transform to identify the instantaneous amplitude and frequency of a signal. This method can be applied to signals that exhibit non-linearity and non-stationarity, but the signal must be oscillatory for the method to work successfully (Huang *et al.* 1998; Huang *et al.* 2008). For a function $x(t)$, the Hilbert transform $c(t)$ is given by

$$c(t) = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}d\tau \tag{4}$$

where $\mathcal{P}$ denotes the Cauchy principal value of the integral, and τ is a dummy integration variable (Huang *et al.* 2008). From this transformation we acquire the function

$$z(t) = x(t) + ic(t) = \alpha(t)e^{i\theta(t)} \tag{5}$$

for an instantaneous amplitude $\alpha(t)$ and an instantaneous phase function $\theta(t)$. From the instantaneous amplitude and IF $\omega(t)$, given by

$$\omega(t) = \frac{d\theta}{dt}, \tag{6}$$

the marginal Hilbert spectrum $h(\omega)$ is created. This portrays the total energy contribution of each frequency with respect to time, and is created by expressing the amplitude as a function of time and frequency $\mathcal{H}(\omega, t)$ (Huang *et al.* 2008)

$$h(\omega) = \frac{1}{T}\int_{0}^{T}\mathcal{H}(\omega, t)dt, \tag{7}$$

where T is the period of time over which the analysis is concerned.

Results

After the removal of atmospheric and zonal tide effects from ΔLOD, a four- to ten-year bandpass filter (Figure 1, left) is used to understand the range of intradecadal signals present (Gillet *et al.* 2022; Rosat *et al.* 2023). To consider the dominant periodic components of ΔLOD, EMD is applied following bandpass filtering to isolate the principal oscillatory mode, IMF 1. The behaviour of IMF 1 in Figure 1a suggests that there are multiple oscillatory periods present within the IMF, and from Figure 1d we observe two main periods which approximately align with those of the SYO and EYO. This is consistent with Figure 1g, which shows the IF fluctuating between periods of 5 and 10 years, as well as highlighting edge effects at either end of the marginal Hilbert spectrum. There is a deviation in the spectra, with a peak in 2014 at an oscillatory period of $\sim$4 years.

The SYO is isolated (Figure 1, middle) using a bandpass filter of 5.40 to 7.51 years, generated using the Rayleigh criterion, to obtain an oscillation with a period of 5.8 $\pm$ 0.8 years. The SYO, as shown in Figure 1b, with the associated periodic content in Figure 1e, shows an interrupted oscillation beginning in 2010. In 2010, the magnitude of the oscillation begins to decrease, and in Figure 1h we see the onset of a spike in the IF and the instantaneous amplitude drop sharply off. A further minima in the IF is then observed, but due to the presence of edge effects, whether this spike is indicative of any behaviour seen in the SYO remains unknown. This suggests that in 2010, the oscillatory period shortens to outside the bandpass filter range.

The EYO is isolated (Figure 1, right) using a bandpass filter of 7.49 to 9.82 years, generated using the Rayleigh criterion, to obtain an oscillation with a period of 9.1 $\pm$ 1.6 years. The EYO signal, as shown in Figure 1c, has a less regular oscillation, which can be associated with a wider spectral window than is contained within the signal (see Figure 1f). In Figure 1c, we observe the magnitude of the signal decrease between 1995 and 2000, with a similar drop in amplitude occurring in the marginal Hilbert spectrum (see Figure 1i), but the magnitude of the signal then recovers.

Discussion

The determined periods of the SYO and EYO in this study are not the expected 5.9-year and 8.5-year periods, but the expected periods do lie within the error range, as given by the Rayleigh criterion. The width of this range is determined by the length of our data series, which is 63 years, and we are unable to differentiate between signals of periods within the error range. Therefore, one of the main limitations of this method for isolation is that, for much finer band limits, further years of expansion of the ΔLOD time series are required.

The trend in the SYO in this study aligns with the observations of Madsen *et al.* (2025), with a breakdown of the signal beginning in 2010, where they determine the subsequent extremum to occur in 2014, giving a 4-year oscillation followed by recovery to a 6-year oscillation, with a shifted phase. We observe a decrease in the instantaneous amplitude and the magnitude of the signal, which both suggest that the energy associated with the oscillation is

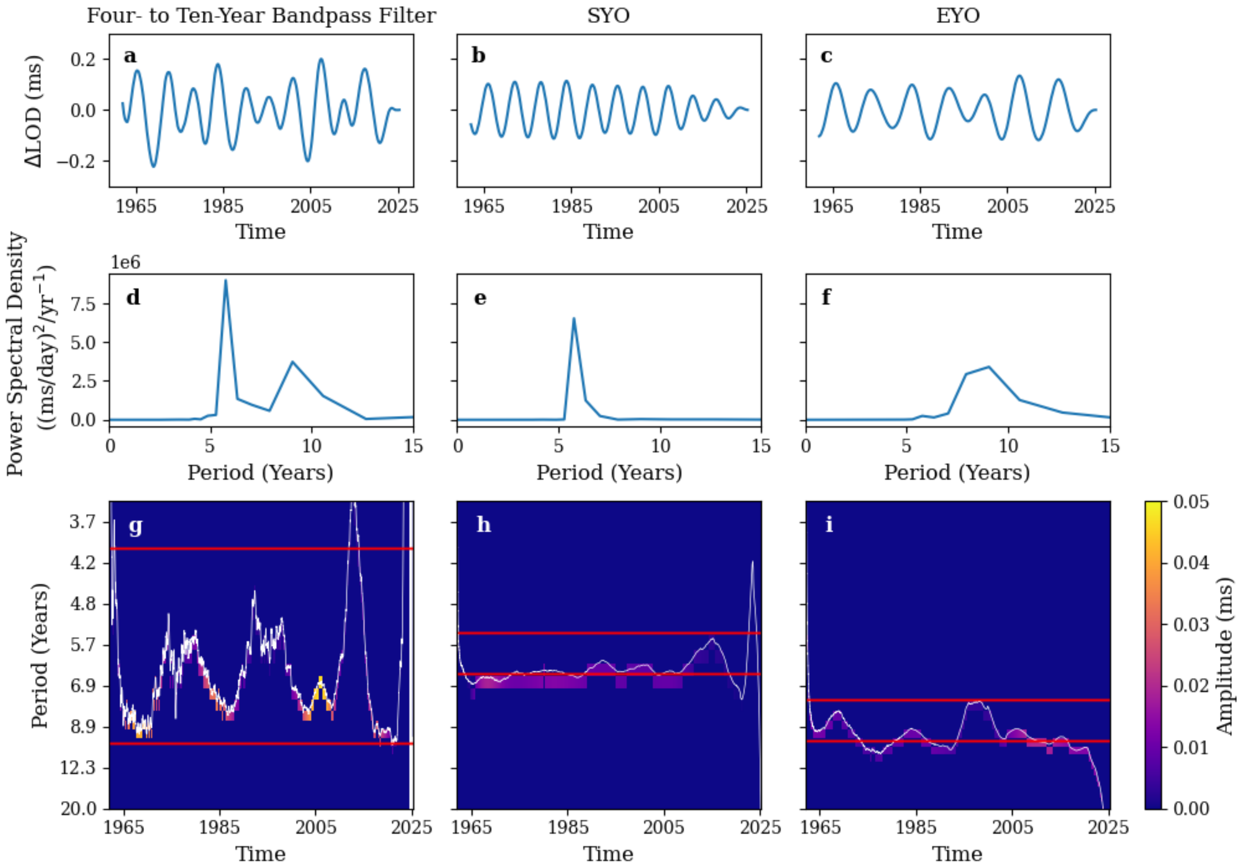

Figure 1: Left column: 4.00–10.00 year bandpass filter on ΔLOD-AT; middle column: 5.40- to 6.51-year bandpass filter on ΔLOD-AT for SYO; right column: 7.49- to 9.82-year bandpass filter on ΔLOD-AT for EYO. (a) IMF 1 of ΔLOD, (b)-(c) timeseries, (d)-(f) periodograms, (g)-(i) marginal Hilbert spectrum. In (g)-(i), the white curve portrays the instantaneous frequency, and the red lines show the upper and lower limits of the bandpass filter.

now no longer within the filter range, supporting a shortened oscillatory period, but no recovery, in the 6-year periodicity of the signal. This likely occurs because the interruption of the 6-year oscillation between 2010 and 2014 is shorter than P_{min}. The reduced period then recovered, but the phase-shifted SYO after 2014 is not resolved by the bandwidth chosen by this study, thus extinguishing the SYO after 2010. This is likely also the phenomenon that leads Duan *et al.* (2020a) to draw the conclusion of a decaying SYO. As such, alternative methods of extracting the SYO have come to different conclusions, suggesting that each method provides different insights into the nature of the SYO.

The activity observed in the EYO shows no decay in magnitude between 2010 and 2025, which indicates that the mechanisms which drove the two oscillations had different sources. Previously, Duan *et al.* (2020a) found the EYO to have a trend of increasing magnitude, whereas we found the trend is broken by an interruption between 1995 and 2000. A phenomenon of low amplitude has previously been observed in the ΔLOD during the 1990s by Holme *et al.* (2013). Holme *et al.* (2013) found this phenomenon arises due to the difficulty in separating anti-correlating intradecadal and interdecadal trends. This could occur in this study due to the wide transition from the passband to the stopband in a first-order Butterworth filter. However, Brown *et al.* (2013) suggest that 1995 to 1998 was a period of frequent jerk activity, and if the EYO and geomagnetic jerks are related, which has been previously proposed by Duan *et al.* (2020a), this may have broken the periodic trend of the signal.

Conclusion

This study investigated intradecadal periods present in the ΔLOD that occur due to angular momentum exchanges as a result of processes in the Earth's core. Initially, the contributions of zonal tides and the Earth's atmosphere are removed, before a first-order Butterworth filter, generated using the Rayleigh criterion, is applied to extract the SYO and EYO. The HHT method is then used to determine the IF of the signal. These oscillations are found to have periods of 5.8 $\pm$ 0.8 and 9.1 $\pm$ 1.6 years, respectively. The SYO shows a decaying trend from 2010 onwards, associated with a shortening in the period of oscillation, where no recovery of the original period of the signal is observed. Furthermore, the EYO breaks down between 1995 and 2000, which may be linked with geomagnetic jerk activity.

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